Reducing Feedback Requirements of the Multiple Weight Opportunistic Beamforming Scheme via Selective Multiuser Diversity

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Abstract—Opportunistic beamforming (OB) relies on the transmission of Channel State Information (CSI) in the form of instantaneous Signal to Noise Ratio (SNR) from Mobile Stations (MSs) back to the Base Station (BS) for scheduling purposes that increase throughput and/or maintain resource allocation fairness. OB is employed in environments of low mobility and low scatter, artificially inducing channel fluctuations that can better exploit Multiuser diversity (MUD). Multiple antennas at the BS are randomly alter the channel's response and generate peaks in gain where users can be scheduled on, maximising system throughput. Additional gains are achieved by transmitting multiple weighting vectors from the BS, but their use can significantly increase the load of the feedback channel and mitigate MUD gains. Selective Multiuser diversity (SMUD) has been proposed for the original beamforming scheme as a technique that reduces feedback requirements substantially without any significant throughput degradations. This paper considers the joined use of multiple weights and SMUD in opportunistic beamforming, aiming to increase capacity while reducing feedback overhead. Results show that with an appropriate use of threshold levels, not only average throughput can be increased through the use of multiple weighting vectors but also a notable decrease in feedback load requirements compared to the conventional OB with SMUD design is achieved.

The terms *Mobile Station (MS) and User* are used interchangeably in this paper, with the same meaning.

I. INTRODUCTION

MUD outlined in [1] is a technique that can be used to exploit channel fading to increase the uplink throughput of a wireless channel. Traditionally, channel fading was considered an undesired feature in wireless channels and channel averaging was used to mitigate its impact. However MUD aims to exploit the inherent variations of a fast fading channel by allowing a user to transmit when its instantaneous channel gain near its peak. Significant capacity increases can be achieved by scheduling the user with the best channel conditions for every time instant. As the number of users in a cell rises, the probability that at least one user has a very strong instantaneous channel increases, allowing a more efficient exploitation of the wireless channel. The performance of MUD in a downlink scenario has been investigated in [2] where similar performance gains to the uplink where observed. MUD, however, fails to provide any clear benefits when the channel does not fluctuate fast enough due to a lack of sufficient strong channel peaks where users can be scheduled on. This scenario can occur in environments of low mobility and low scatter.

OB has been presented in [3] as a method to allow exploitation of MUD even in slow fading environments. This is achieved by employing multiple antennas at the BS to artificially induce random channel fluctuations generating more distinct peaks where users can be scheduled on.

Exploiting MUD efficiently imposes tight requirements on feedback. Each user is required to transmit CSI via a feedback channel back to the BS for every time slot. Overall capacity gain is achieved by determining the user with the best CSI for every time slot and allocating all resources to that user. The amount of CSI fed back to the BS determines the extent to which efficient scheduling can be used. Transmission of the achievable data rate can be used as a good measure of the channel quality of each user. Theory suggest than an infinite rate feedback is required. In practice however, feedback rate is quantised. Depending on the quantisation of this value the feedback load varies, but still a significant load in the feedback channel is imposed, arising from the requirement to transmit real valued terms for all active MSs for every time slot.

In [4] it has been argued that transmitting data rate values for all users conveys redundant information, and that the same throughput performance can be achieved by allowing only users with strong instantaneous channels to feed back information. It can be argued that even if a Proportionally Fair (PF) scheduling algorithm is employed, where a user is scheduled based not only on its instantaneous channel conditions but also on its total throughput measured over a weighted window length t_c [3], only users with relatively strong channels have any realistic chance of being scheduled for transmission in the next time slot. Hence CSI from users with weak instantaneous channels may in fact be omitted, without any impact on the overall throughput performance. The proposed SMUD approach allows users to feed back CSI only if their instantaneous channel gain is above a predefined threshold level. Results in [4] show that the capacity of a system employing OB can be maintained by only allowing the strongest 10% of users to be considered for scheduling.

When the number of active users is low, MUD gives reduced gain, even if OB is employed, due to the reduced probability of the system in locating users with good instantaneous channel conditions. To overcome the problem of reduced diversity, the transmission of multiple weighting vectors from the BS is proposed in [5]. Results show that significant gains can be achieved with this beamforming configuration, especially when the number of users in the system is small. However this increase in

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downlink throughput occurs at the expense of increased feedback, as each MS is required to feed back data rate values for each of the weighting vectors back to the BS.

In this paper the operation of a reduced feedback Multiple Weighting Vector Opportunistic Beamforming based on SMUD is developed and analysed. The proposed scheme is compared in terms of achievable throughput and feedback load overhead to a full CSI scheme with conventional OB and multiple weighting OB (MWOB) as well as with the SMUD scheme for conventional OB

II. SYSTEM MODEL

This paper considers a configuration of a single BS with 8 transmit antennas and K single antenna mobile stations. Simulations have shown that full spatial diversity can be achieved with a minimum of 8 transmit antennas. By increasing the number of transmit antennas beyond this point channel hardening effects begin to emerge, that result in no further increase in downlink throughput In [6] it was shown that employing antennas in excess of this number will incur only additional hardware costs without any clear throughput benefits. Transmit antennas are not correlated and channel gain vectors across different users are independent. Consideration is given only to the downlink scenario; consideration of the uplink scenario is a straightforward extension. Unless otherwise stated, a homogeneous network is assumed, where all users have the same average SNR = 0dB. A greedy scheduling approach, that essentially selects the user with the strongest instantaneous channel conditions, is employed where appropriate. Since it is assumed that all users have uniform channel conditions the PF with this specific time scale ensures both fairness and optimum rate growth. For each user k, the channel gain vector h_k is assumed to be perfectly known at the receiver, but not necessarily known to the BS. Slow fading is assumed throughout this paper where the channel remains constant for several time slots. Simulations in this paper assume that channel remains constant for 1000 time realisations; a good assumption for slow fading. For every user a low-rate error-free feedback channel exists that conveys CSI back to the BS. The overhead associated with the uplink feedback channel does not directly affect throughput calculations, but will be calculated in order to determine feedback requirements for each scheme. The overheads on the downlink slot associated with the transmission of multiple random weighting vectors from the BS in OB *are* considered and simulation results do take into account the loss in throughput caused by the transmission of these vectors.

III. MULTIPLE WEIGHT OPPORTUNISTIC BEAMFORMING

In the original OB scheme proposed by Viswanath [3], a known training sequence $x(t)$ is transmitted from the BS multiplied by a random weighting vector $v(t)=(v_1(t),....v_N(t))^T$ to determine the instantaneous channel conditions of each user. In particular $x(t)$ is multiplied by a random complex number $\sqrt{\alpha_n(t)}e^{j\theta_n(t)}$ at antenna n, for $n=1,....,N$, where the terms α and θ represent random amplitude and phase variations. Total power is preserved by setting $\sum_{n=1}^{N}\alpha_n(t)=1$. Defining $h_{nk}(t)$ as the complex channel gain from the *n-th* antenna to user k at time slot t, the overall channel "seen" by user k can be written as:

$$H_k(t)=\sum_{n=1}^{N}\sqrt{\alpha_n(t)}e^{j\theta_n(t)}h_{nk}(t) \tag{1}$$

The physical channel gain vector of the k_{th} user in time slot t can be defined as $h_k(t)=(h_{1k}(t),.....,h_{Nk}(t))^T$. The term $h_{nk}(t)$ is the complex channel gain from antenna n to the k_{th} user and it is modelled as s complex Gaussian random variable with zero mean. In a slow fading scenario the term $h_{nk}(t)$ remains constant for the number of time slots defined in Section II. From Eq. 1 it can be seen that now the channel changes for every slot t since the channel term is multiplied by a random complex number.

To calculate the instantaneous channel gain $|H_k(t)|^2$ of each user, the BS transmits a known training sequence which is multiplied by the random weighting vector $w(t)=(w_1(t),......,w_N(t))^T$, which is comprised of the random phase and amplitude variations from each transmit antenna. The resulting requested sum rate of each user is then fed back to the BS. According to Shannon's capacity theorem [7], the requested rate is given by (P_k being the instantaneous power available to user k):

$$R_k[t]=W\log_2\left(1+\frac{P_k|H_k(t)|^2}{N_0}\right) \tag{2}$$

The BS collects all the requested rates and schedules for transmission the best user using a scheduling algorithm (e.g. round robin, maximum SNR or PF).

The MWOB scheme suggests that instead of multiplying the complex channel gain $h_{nk}(t)$ of each user by just one weighting vector, additional diversity gains can be accomplished by considering Q different random weighting vectors for each user. Essentially this creates Q different channel realisations for each MS, increasing the multiuser diversity order from $\log\log(K)$ to $\log\log(QK)$, an approach especially suitable when a low number of active users are present in the system.

The MWOB scheme proposed in [5] states that each user needs to transmit back channel gain values arising from the Q different weighting vectors. The decision on the weight that returns the maximum gain for each user is evaluated at the BS. Essentially this constitutes a *Q-fold* increase in feedback

The transmission of a weighting vector occupies a certain fraction of the downlink time slot. This finite time is called a minislot. The minislot can be placed at the end of the previous downlink slot and given that channel statistics do not significantly deviate between adjacent slots, they can be used as a measure to determine channel gains of the next slot. Given that Q vectors are transmitted, the BS determines the optimum weighting vector $q^{opt}(t)$ according to:

$$q^{opt}(t)=\arg\max_{q=1,...Q}\left(\max_{k=1...K}R_{q,k}(t)\right) \tag{3}$$

where $R_{q,k}$ is the theoretical achievable rate from the assignment of the *q-th* weighting vector to the *k-th* user.

The transmission of Q weights per slot results in an increased overhead load on the downlink slot, since the available length for useful transmission of data is reduced as the number of weighting vectors increases. Defining L and τ as the length of the total downlink time slot and minislot respectively, the maximum throughput when considering degradations due to minislot overhead is defined as [5]:

$$T_Q(t) = (L - \tau Q)\left(\max_{q=1,....Q,k=1.....K} R_{q,k}(t)\right) \tag{4}$$

From Eq. 4, it can be seen that the transmission of a weighting vector limits the duration on each time slot for useful data transmission. The use of a large number of weighting vectors will result in a very large overhead that will significantly hinder throughput performance. On the other hand it is necessary to have sufficient vectors to be able to extract increased diversity gains. An optimum number of weights Q_{opt} that achieves the best tradeoff between diversity gains and slot overhead should exist. Eq. 4 can be re-written using Euler's approximation to harmonic series from which the optimum number of weights for a given number of active users and minislot length can be calculated. The term γ corresponds to Euler's constant [5].

$$T_Q(t) = \left(1 - \frac{\tau}{L}Q\right)(\log(QK + \gamma)) \tag{5}$$

From Equation (5) the optimum number of weighting vectors can be calculated given the number of users K in the system and the ratio of the minislot length over the total length of the downlink slot $\frac{\tau}{L}$. Typical values of $\frac{\tau}{L}$ lie in the region of 5-10% [8, 9]. As the number of users in the systems grows, inherent MUD increases proportionally, diminishing the need of multiple weights, that have the disadvantage of increased feedback overhead. Figure 1 shows how Q_{opt} varies for an increasing number of users for two different minislot lengths, $\frac{\tau}{L} = .05$ and $\frac{\tau}{L} = 0.1$. It can be seen that when the length of a minislot is large, a more conservative approach towards using multiple weighting vectors is required, due to the significantly higher resulting overhead that mitigates any diversity gains obtained by multiple weighting vector transmission.

Figure 2 presents throughput results for different numbers of users obtained by Monte Carlo simulations. For reference, the performance of the conventional OB scheme (*Q=1*) is also shown for the two minislot lengths under consideration. It can be seen that significant gains can be extracted through the use of multiple weighting vectors. The abrupt steps evident in some curves can be attributed to the discrete variation of Q_{opt}. The expected loss in throughput due to the increased overhead from the multiple weight vectors has been outweighed by the enhanced diversity gains that these weights give.

As mentioned earlier, the improved throughput of the MWOB comes at the expense of a Q_{opt}*-fold* increase in feedback. Such an increase can severely degrade the performance of the system. In the following section multiple weighting vector scheme employing Selective Multiuser Diversity is proposed, in order to significantly reduce the amount of the required fed back CSI, whilst maintaining comparable downlink rates to the full MWOB SNR feedback scheme.

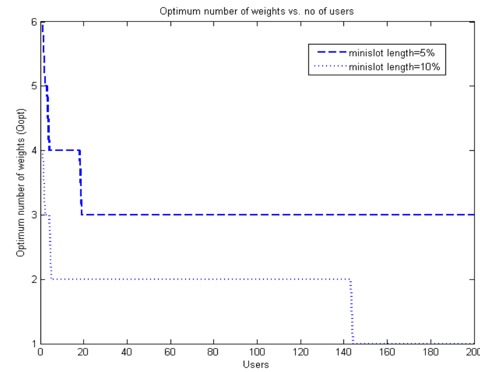

Figure 1: Optimum number of weights vs. number of users

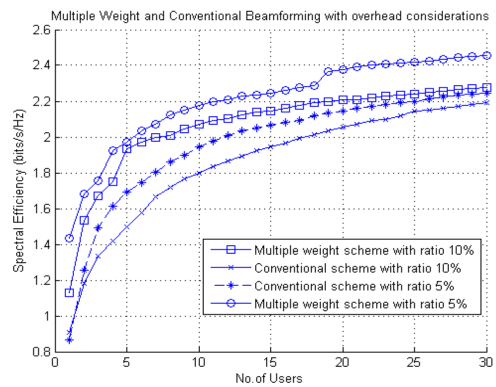

Figure 2: Multiple Weight and Conventional Opportunistic Beamforming

IV. REDUCING FEEDBACK REQUIREMENTS OF THE MULTIPLE WEIGHTING OB SCHEME

A. Conventional Opportunistic Beamforming with SMUD

Selective Multiuser Diversity (SMUD) has been proposed in [4] as a scheme that can significantly reduce the feedback requirements of the conventional OB scheme by allowing users to transmit CSI only if their SNR exceeds a certain threshold level which is set by the BS. No significant loss in throughput occurs because even in the case where a PF scheduling algorithm with a finite window length is employed, only users that have channels near their peak have any realistic possibility of being scheduled for transmission. Therefore, CSI from weak users is redundant and only results in increasing the load of the feedback channel.

In this paper, a MWOB scheme with SMUD is considered in order to reduce the feedback load, whilst aiming to maintain rates comparable to the full SNR feedback MWOB scheme, which results in a *Q-fold* increase in feedback requirements. This is due to the fact that each MS is required to transmit SNR information back to the BS for each of the Q transmitted vectors. SMUD can be used to reduce this prohibitive feedback load by restricting weak channel gains for each user from being fed back to the BS.

The normalised feedback load $\overline{F}$ is defined as the ratio of users $P(t)$ above the threshold at slot t over the total number of users in the system K. Initially assuming a single weighting vector transmission, the normalised feedback load is given by:

$$\overline{F} = \frac{P(t)}{K} \tag{6}$$

The number of users whose instantaneous SNR is above the threshold is defined as $P(t) = card$ [k, such that $\gamma_k(t) \geq \gamma_{th}$], where $\gamma_k(t)$ is the instantaneous SNR value of user k in slot t, γ_{th} being the threshold level and *card* being the cardinal operator. For an i.i.d Rayleigh fading scenario with an average SNR = $\overline{\gamma}$, the normalised load equation can be expressed as [4]:

$$\overline{F} = e^{-\gamma_{th}/\overline{\gamma}} \tag{7}$$

It is possible that for some time slots the BS cannot identify any eligible users to be considered for transmission. This happens in the case where no user has an instantaneous SNR higher than the threshold. The number of fed back users, P(t) is zero in this case. This condition is defined as outage. The outage probability is therefore equal to the probability that no user in a slot t has an instantaneous SNR above the threshold.

$$P_o = \text{Prob}\left(\gamma_k(t) < \gamma_{th}\right), \text{ for all } k=1\ldots K \tag{8}$$

When the system is in outage, the BS has no information regarding the instantaneous channel conditions of any of the MSs. In this case the BS can continue transmission to the user selected in the previous slot, or can blindly assign resources to a MS. For an i.i.d Rayleigh fading scenario the outage probability can be written as:

$$P_o = \left(1 - e^{-\gamma_{th}/\overline{\gamma}}\right)^K \tag{9}$$

B. Integration of SMUD in MWOB

In Figure 3 the throughput of the MWOB and the conventional OB scheme combined with SMUD for different threshold levels is compared with the full SNR feedback MWOB scheme, for a minislot ratio of 5%. Calculations take into account throughput reduction due to the overhead caused by the transmission of multiple weighting vectors on the downlink slot according to Eq. 4. Simulations show that MWOB capacity can be maintained even by only allowing SNR values to be fed back to the BS for vectors that return a channel gain 5dB higher to the mean for the MWOB scheme, whereas for the conventional OB scheme a threshold level of 3dB is required to maintain capacity. As the threshold level increases the average sum rate drops. This is due to the fact that the outage probability increases leading to completely random resource assignments by the BS, arising from the complete lack of of instantaneous channel. The random assignment of users in this scenario can help to minimise throughput losses, but not to an extent where the benefits of feedback load reduction from the high threshold level would outweigh the throughput degradations caused by the increased threshold level. Depending on the requirements of the system, the threshold level can be altered to either maximise downlink throughput by setting a relatively low threshold, or achieve significant feedback reduction by setting a high threshold level.

It is interesting to observe the variation of the optimum threshold level between the conventional and the multiple weighting vector scenarios. The MWOB can tolerate a much higher threshold level (5dB) due the higher resulting channel gains, whereas the conventional OB scheme requires a significantly lower threshold (3dB) in order to minimise excessive outage instants.

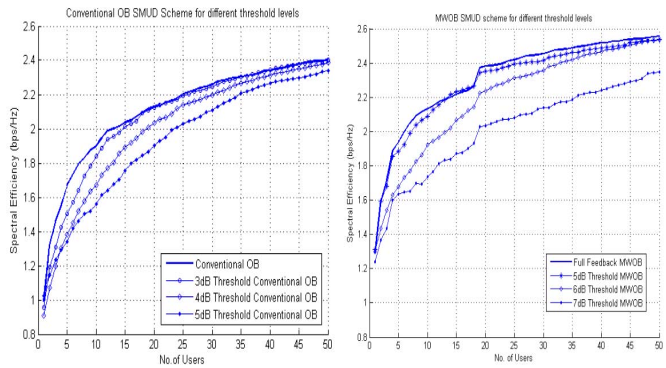

Figure 3: Multiple Weighting Vector Opportunistic Beamforming with SMUD

C. Outage Probability for the joined SMUD MWOB scheme

The outage probability for the joined SMUD MWOB scheme is defined as the event where the number of eligible channels fed back to the BS is zero, i.e. $P_o = Prob\ (Q_{opt}C(t)) = 0)$. This event corresponds to the probability that none of the random weighting vectors generate a channel realisation with gain higher than the predefined threshold, for any of the available users. This probability is given by:

$$P_o = Prob\left(\gamma_k(t) < \gamma_{th}\right) \textit{ for all } k=1\ldots..Q_{opt}.K \tag{10}$$

In Figure 4 the scheduling outage probability of the two schemes under consideration is examined as a function of the threshold level in dB for different numbers of active users, for an average SNR = 0dB. It can be observed that for both schemes outage is reduced for an increasing number of users, as it would be expected. Considering the curves for 50 users, the outage probabilities for the corresponding optimum threshold are very small. Thus, the convergence of the schemes to a full CSI feedback configuration.

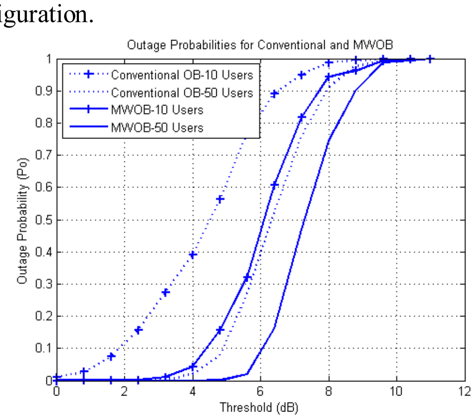

Figure 4: Outage Probability of joined Conventional and MWOB SMUD scheme

D. Feedback overhead comparison

The SMUD-MWOB scheme can tolerate a higher threshold level without significant throughput losses. In order to investigate the effect that this higher threshold level has on the feedback overhead, the expected normalized feedback load for conventional and MWOB is examined as a function of the threshold level. Each random weighting vector consists of i.i.d complex numbers, multiplying each user's instantaneous channel and in MWOB generates Q different, independent channel realisations, denoted by $C(t)$, for each user. Hence (6) can be extended to accommodate for the probability of having several channel realisations for each user that fulfil the eligibility criterion for transmission, rather than a single one. For the conventional OB case ($Q=1$) the terms $C(t)$ and $P(t)$ coincide. In a multiple weighting vector implementation however, the number of eligible channel realisations will on average be Q_{opt} times larger, as the number of channel realisations increases by this factor. Hence the normalised load for MWOB can be defined as:

$$\overline{F} = \frac{Q_{opt} P(t)}{K} \tag{11}$$

Figure 5 shows the resulting feedback load of the two schemes under consideration as a percentage of the full SNR scheme with a single weighing vector transmission. Analysis has been concentrated in the 0dB SNR region. Q_{opt} is set to 3, which holds true for a wide range of active users, for a minislot length of 5%. Results in Figure 3 have identified the threshold levels that give minimal throughput losses for conventional and MWOB at 3dB and 5dB respectively. This threshold levels correspond to a normalized feedback load value of 13.6% for conventional beamforming and 12.7% for the MWOB scheme.

Considering the fact that the transmission of multiple weights in a full feedback scheme results n a Q-fold increase in feedback overhead, the resulting feedback overheads with SMUD are extremely encouraging. The MWOB scheme with SMUD has not only managed increases in downlink throughput gains through increased diversity, but has resulted in a further reduction in feedback overhead, which was the fundamental drawback of a full feedback MWOB scheme.

V. CONCLUSIONS

In this paper a scheme that considers Selective Multiuser Diversity scheme with multiple weighting vector, opportunistic beamforming has been proposed. Throughput simulation results have indicated that most of the gains arising from increased diversity gains via multiple weighting vector transmission can be maintained by SMUD through appropriate threshold selection. It has been shown how the threshold for MWOB can be increased with respect to conventional beamforming without any significant deviations in outage. The increased threshold level does not only manage to preserve capacity, but also results in better user selectivity, reducing the feedback load even further than SMUD achieves with conventional beamforming. This result is quite significant, since the main drawback of a full feedback MWOB scheme results in a *Q-fold* feedback overhead increase.

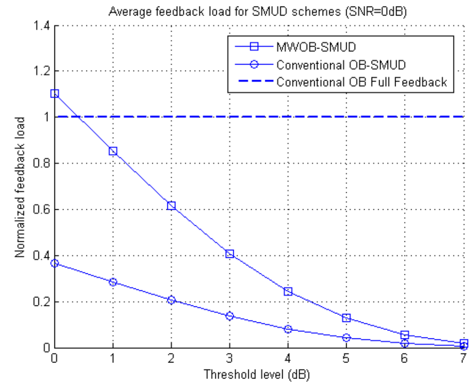

Figure 5: Average feedback load vs. threshold for the SMUD scheme

VI. ACKNOLEDGEMENTS

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